Isogeometric Mass Matrix preconditioner

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ABSTRACT

Linear systems associated with the Galerkin mass matrix arise in some relevant cases, for example: in finite difference discretization of the time variable, coupled with Galerkin discretization in space; in the evaluation of an L^2 -projection or when the mass matrix is used as a smoother in a multigrid method, see [5]. In this talk we propose an efficient preconditioner for isogeometric mass matrix associated with a variety of geometries. The main idea is to use, as a preconditioner, a diagonally scaled parametric mass matrix, which turns out to be Kronecker product of univariate mass matrices. Thanks to the Kronecker structure, the application of the preconditioner is fast. For the multipatch case, we use the single patch strategy as local solver for the Additive Schwarz Preconditioner. Under the assumption of regular parametrizations, we prove that our preconditioner is robust with respect to the mesh refinement. In particular, for single patch domains, it can be proved that the condition number of the preconditioned matrix goes to one as the mesh size goes to zero. Finally we present numerical examples that includes: single patch, multipatch and trimmed geometries for bidimensional and three-dimensional cases, with both regular and singular parametrizations. This tests show that the preconditioner is also robust with respect to the spline polynomial degree.

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