Continuity constraints for phase-fields on deforming multi-patch shells

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ABSTRACT

This work presents numerical techniques to enforce continuity constraints on multi-patch surfaces for three distinct problem classes. The first are thin-shells described by Kirchhoff-Love kinematics. Their governing equation is a vector-valued, fourth-order, non-linear, partial differential equation (PDE) that requires at least C^1-continuity within a displacement-based finite element formulation. The second class are phase separations modeled by a phase-field. Their governing equation is the Cahn-Hilliard equation – a scalar, fourth-order, non-linear PDE – that may be coupled to the thin-shell PDE. The third class are brittle fracture processes modeled by a phase-field. These are commonly described by a scalar, fourth-order, non-linear PDE that is similar to the Cahn-Hilliard equation and may also be coupled to the thin-shell PDE. Using a direct finite element discretization, the two phase-field equations also require at least a C^1-continuous formulation. Isogeometric surface discretizations – often composed of multiple patches – thus require a constraint that enforces the C^1-continuity of displacement and phase-field. For this, two numerical strategies are presented: A Lagrange multiplier formulation and a penalty regularization. They are both implemented within the curvilinear shell and phase-field formulation of [1,2] and illustrated by several numerical examples. These consider deforming shells, phase-separations on evolving surfaces, and dynamic fracture.

REFERENCES

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