Spline spaces provide more accuracy per degree of freedom

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ABSTRACT

Since the inception of IGA [1, 2], it was observed that the solution of the PDE is well approximated using fewer degrees of freedom compared to more other methods such as FEM and DG. The presentation will outline how the available [3]worst-case-approximation-estimates for piecewise polynomial spaces on uniform partitions of $[0,1]$ give a theoretical proof of the observed advantage.

More precisely, if $\mathbb{S}_{p,k,n}$ is the space of globally $\mathcal{C}^k$ piecewise polynomials of degree p on n uniform segments and $C_{p,k,n}$ is the smallest constant such that:

$$\inf_{g\in\mathbb{S}_{p,k,n}} \|f-g\|_{L^2} \leq C_{p,k,n}\|\partial^{p+1} f\|_{L^2}, \qquad \forall f \in H^{p+1}(0,1)$$

then for $p>2$, sufficiently large n, and m such that $\dim\mathbb{S}_{p,k,n} = \dim\mathbb{S}_{p,p-1,m}$ it holds

$$C_{p,p-1,m} < C_{p,k,n}.$$

Similar results hold for lower order Hilbert spaces and broken-Hilbert spaces.

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