IsoGeometric Analysis and Fourier basis for problems in axisymmetric domains

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ABSTRACT

Problems defined on axisymmetric domains have a high practical relevance, in particular for applications to particle accelerators [2]. In this context, it is natural to try to exploit the symmetry of the geometry by means of cylindrical coordinates and the Fourier basis. This often allows to decouple the full three-dimensional problem, defined on the whole volume, in a sequence of two-dimensional ones, defined on the cross-section, with significant advantage in terms of computational cost. On the other hand, it is well known that discretization that properly fulfil the discrete de Rham sequence are required to produce correct results, see [3] for an example in Cartesian coordinates using IGA or [1]. It is therefore natural to combine the two aspects and define proper discretizations for the two-dimensional problems [4]. In this work, we extend the results presented in [4] employing the IGA paradigm to define the geometry of the cross-section and to define the discrete spaces. This allows to exploit the advantages of IGA and to define higher order methods. We then present some numerical results on test cases in the electromagnetic framework.

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