

# A Mathematical Study of Scaled Boundary Isogeometric Analysis

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## ABSTRACT

This contribution is concerned with scaled boundary parametrizations for Isogeometric Analysis (IGA). These are easy to construct and advantageous if only a boundary description of the computational domain is available and the domain is star-shaped. The idea goes back to the Scaled Boundary Finite Element Method [3], which has then been extended to IGA. We study here these parametrizations as bivariate or trivariate B-spline functions that are directly suitable for standard Galerkin-based IGA.

In the talk, the relation of this approach to the classical concept of Isogeometric Analysis is analyzed. In particular, focusing on Poisson's equation as model problem, we compare classical IGA with scaled boundary IGA where the weak form and Galerkin projection are used both in scaling and in circumferential direction. Furthermore, we explain the relation between these two methods by means of the Laplace-Beltrami operator and show their equivalence. Additionally the singularity in the scaling center is investigated with regard to the implications for the weak form and for the numerical integration. It turns out that by eliminating the superfluous degrees of freedom associated with the multiple knot in the scaling center, the weak form stays well-defined. We also comment on extensions of the approach to the multipatch framework and on criteria for the choice of the scaling center.

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## REFERENCES

- [1] C. Arioli, A. Shamanskiy, S. Klinkel and B. Simeon, *Scaled Boundary Parametrizations in Isogeometric Analysis*. Computer Methods in Applied Mechanics and Engineering 349, 576–594, 2019.
- [2] L. Chen, B. Simeon and S. Klinkel, *A NURBS based Galerkin approach for the analysis of solids in boundary representation*. Computer Methods in Applied Mechanics and Engineering 305, 777–805, 2016.
- [3] C. Song and J.P. Wolf, *The scaled boundary finite-element method - alias consistent infinitesimal finite-element cell method - for elastodynamics*. Computer Methods in Applied Mechanics and Engineering 147, 329–355, 1997.