A posteriori error estimation and convergence of adaptive isogeometric methods

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ABSTRACT

The CAD standard for spline representation in 2D or 3D relies on tensor-product splines. To allow for adaptive refinement, several extensions have emerged, e.g., analysis-suitable T-splines, (truncated) hierarchical B-splines, or LR-splines.

In our work [1], we derived a set of abstract properties for finite element ansatz spaces, the underlying meshes, and the mesh-refinement that automatically guarantee reliability and efficiency of a standard weighted-residual error estimator. When a standard adaptive mesh-refining algorithm is used, these properties imply convergence of the so-called total error (i.e., energy error plus data oscillations) at optimal algebraic rate with respect to the number of degrees of freedom. So far, we have verified that hierarchical B-splines on certain admissible hierarchical meshes as well as analysis-suitable T-splines on admissible T-meshes in the sense of [2] fit into the developed abstract framework. Moreover, for truncated hierarchical B-splines, the properties have been implicitly and independently shown in [3].

Usually, CAD provides only a parametrization of the boundary $\partial\Omega$ instead of the domain Ω itself. The boundary element method (BEM) circumvents this difficulty by working only on the CAD provided boundary mesh. In [4], we developed a similar abstract framework for BEM as in [1] that guarantees reliability of a standard weighted-residual estimator as well as convergence at optimal rate of the latter. Again, the introduced properties have been verified for (truncated) hierarchical B-splines as well as analysis-suitable T-splines. Recently, in [5], we showed that splines on a one-dimensional boundary with an adaptive algorithm that does not only steer the mesh-refinement but also the local smoothness of the ansatz functions satisfy these properties.

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