Efficient Integration Schemes in NURBS-based Isogeometric Analysis – Comparison and Application to Shell Elements

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ABSTRACT

The efficiency of the NURBS-based isogeometric Galerkin method highly depends on the chosen numerical integration scheme for the evaluation of the stiffness matrix and the residual vector. In an efficient implementation, the computational costs for the formation of the stiffness matrix scale almost linearly with the total number of integration points. Furthermore, also the accuracy of the solution depends on the chosen integration scheme, since spurious locking effects increase with the number of integration points. Gauss integration is probably the most common approach for numerical integration in NURBS-based IGA, whereby the higher continuity in IGA can be exploited for a reduction of the number of integration points, see [2]. More sophisticated integration schemes have been proposed in [3,4], where a nonlinear equation is solved in order to compute integration weights and quadrature points. Using the minimum possible number of quadrature points, the nonlinear equation aims at integrating the univariate B-spline spaces exactly. Thus, the methods can be considered as optimal.

In this contribution, an implementation of the optimal integration method [3] for general NURBS surfaces with non-uniform knot vectors is presented. The integration method is applied to the Poisson problem, two-dimensional plane stress problems and isogeometric shell elements as proposed in [5]. The numerical examples of this study focus on the accuracy of the integration for NURBS surfaces with non-uniform weights. A further very important aspect is the comparison of computational costs between the different methods. The alleviation of locking effects, which comes along with a reduction of the number of integration points, is studied for the shell elements proposed in [5]. Furthermore, a comparison to the locking-free formulation of [6] is given.

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