Approximation with Maximally Smooth Splines and Sharp Error Estimates

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ABSTRACT

Splines are piecewise polynomial functions that are glued together in a certain smooth way. When using them in an approximation method, the availability of sharp error estimates is of utmost importance. Classical error estimates in Sobolev (semi-)norms for spline approximation are expressed in terms of

(a) a certain power of the maximal grid spacing, the so-called approximation power,

(b) an appropriate derivative of the function to be approximated, and

(c) a "constant" which is independent of the previous quantities but usually depends on the spline degree.

An explicit expression of the constant in (c) is not always available in the literature, because it is a minor issue in the most standard approximation analysis; the latter is mainly interested in the approximation power of spline spaces of a given degree [1].

These estimates are perfectly suited to study approximation under h-refinement, i.e., refining the mesh. On the other hand, one of the most interesting features in isogeometric analysis is k-refinement, i.e., degree elevation with increasing interelement smoothness. The above mentioned error estimates are not sufficient to explain the benefits of approximation under k-refinement as long as it is not well understood how the degree of the spline affects the whole estimate, including the "constant" in (c).

In this talk we focus on a priori error estimates with explicit constants for approximation by spline functions defined on arbitrary knot sequences and maximal smoothness, in both the periodic and the non-periodic cases [2]. These a priori estimates are actually good enough to cover convergence to eigenfunctions of classical differential operators under k-refinement.

REFERENCES

[1] Schumaker, L. L. *Spline Functions: Basic Theory, 3rd edn*. Cambridge University Press (2007).

[2] Sande, E., Manni, C., and Speleers, H. Sharp error estimates for spline approximation: Explicit constants, n-widths, and eigenfunction convergence. *Mathematical Models and Methods in Applied Sciences*, to appear.