Efficient p-multigrid based solvers for Isogeometric Analysis

Roel Tielen[†*], **Matthias Möller**[†] **and Kees Vuik**[†]

[†] Delft Institute of Applied Mathematics (DIAM)
Delft University of Technology
Van Mourik Broekmanweg 6, 2628 XE Delft, the Netherlands
e-mail: r.p.w.m.tielen@tudelft.nl, web page: http://ta.twi.tudelft.nl/nw/users/rtielen/

ABSTRACT

Over the years, Isogeometric Analysis (IgA) [1] has shown to be a successful alternative to the Finite Element Method, both in academia and industrial applications. However, solving the resulting linear systems efficiently remains a challenging task. For instance, the condition number of the stiffness matrix scales quadratically with the mesh width h, but, in contrast to standard Finite Elements, exponentially with the order of the approximation p [2]. The performance of (standard) iterative solvers thus decreases fast for larger values of p.

Recently, the authors presented a solution strategy for IgA discretizations that is based on p-multigrid techniques [3]. This approach makes use of a hierarchy of B-spline based discretizations of different approximation orders, where the 'coarse grid' correction is determined at level $p = 1$, enabling us to use well established solution techniques developed for low-order Lagrange finite elements. The use of a smoother based on an ILUT factorization within the p-multigrid method has shown to lead to convergence rates independent of h and p [4].

In this talk, we present numerical results for different benchmark problems on non-trivial (multipatch) geometries [5]. It follows from a spectral analysis, that the asymptotic convergence rate of the p-multigrid method is (essentially) independent h and p, but slightly depends on the number of patches. For multipatch geometries, in which the resulting system matrix has a block structure, we consider the use of block ILUT. Results indicate that the use of block ILUT as a smoother can be an efficient alternative to ILUT on multipatch geometries within a heterogeneous HPC framework.

REFERENCES

[1] T.J.R. Hughes, J.A. Cottrell and Y. Bazilevs, *Isogeometric Analysis: CAD, Finite Elements,NURBS, Exact Geometry and Mesh Refinement*, Computer Methods in Applied Mechanics and Engineering, 194, 4135 - 4195, 2005

[2] K.P.S. Gahalaut, J.K. Kraus, S.K. Tomar. *Multigrid methods for isogeometric discretization*, Computer Methods in Applied Mechanics and Engineering, 253, 413 - 425, 2013

[3] R. Tielen, M. Möller and C. Vuik, *Efficient multigrid based solvers for Isogeometric Analysis.* Proceedings of the 6th European Conference on Computational Mechanics and the 7th European Conference on Computational Fluid Dynamics, Glasgow, UK, 2018

[4] R.Tielen, M. Möller, D. Göddeke and C. Vuik, *Efficient p-Multigrid Methods for Isogeometric Analysis*, arXiv: 1901.01685.

[5] R. Tielen, M. Möller and C. Vuik, *Efficient p-Multigrid based solvers for multi-patch geometries in Isogeometric Analysis*, Proceedings of the 3rd Conference on Isogeometric Analysis and Applications (IGAA 2018), 23-27 April 2018, Delft, The Netherlands