Shape Sensitivity Analysis for Hyperbolic Systems using an Isogeometric Discontinuous Galerkin Method

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ABSTRACT

The framework of this work is the Isogeometric Discontinuous Galerkin (IDG) method, presented in [1], which relies on the coupling of a Discontinuous Galerkin formulation and NURBS representations. This methodology allows to solve partial differential equations with an exact representation of boundary shapes thanks to the Isogeometric Analysis (IGA) paradigm [2], while using a DG formulation well suited to hyperbolic systems and discontinuity capturing.

Sensitivity analysis consists in evaluating the derivatives of the solution fields with respect to a given parameter, in the perspective of optimization or uncertainty quantification for instance. When this parameter controls the geometry, the sensitivity analysis becomes tedious, because it requires the knowledge of high-accurate solution derivatives at the domain boundary [3]. Therefore, the objective of this work is to use the properties of exact shape representations and high accuracy of IDG to provide an efficient sensitivity analysis methodology for hyperbolic systems.

As case study, we consider the inviscid compressible flow between two cylinders, with the internal radius as shape parameter. In particular, we compare the accuracy of the results obtained by using an exact representation of the boundary shape and a classical piecewise linear approximation of the geometry.

Finally, we consider an alternate methodology to perform sensitivity analysis for shape parameters. It relies on the use of the map between the physical domain and a parametric domain, the latter one being independent from the shape parameters as described in [4]. The natural mapping provided by the NURBS representation allows to derive this methodology in a straightforward and elegant way.

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