

# $G^1$ -conforming quadrilateral and triangular finite elements for the analysis of Kirchhoff plate

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## ABSTRACT

A family of quadrilateral and triangular  $G^1$ -conforming finite elements, based on the Gregory patch [1], suitable for the analysis of the Kirchhoff plate model are analyzed. These elements present cubic control of the normal derivative along the sides and therefore be combined among themselves and with side beams [3, 2, 4]. The coupling strategy are based on the formulas of the Bézier degree elevation. Using the Bézier extraction,  $G^1$ -conforming hierarchic refinement can be designed for these elements.

A rational enhancement of the base-polynomial spaces, known in the CAGD-literature as Gregory Patch, useful to design  $G^1$ -conforming elements on general  $C^0$ -conforming unstructured meshes is presented. The second derivatives at the corners of the rational approximant present finite discontinuities, that prevent the elements from passing the bending patch test. These discontinuities are removed using a constrained version of the Gregory Patch, obtained imposing their suppression via Lagrange multipliers. This strategy is indicated by  $CG^1$ -formulation.

In this way, the rational conforming space collapses into a conforming rearrangement of the original polynomial space. The proposed  $CG^1$ -formulation design triangular and/or quadrilateral elements that pass the bending patch test, present optimal rate of convergence on general unstructured meshes and are easy to assemble together.

## REFERENCES

- [1] Gregory, J.A. Smooth interpolation without twist constraints *Computer Aided Geometric Design* (1974) :71–87.
- [2] Greco, L. and Cuomo, M. and Contrafatto L. A quadrilateral  $G^1$ -conforming finite element for the Kirchhoff plate model *Computer Methods in Applied Mechanics and Engineering* (2019) **346**:913–951.
- [3] Cuomo, M. and Greco, L. An implicit strong  $G^1$ -conforming formulation for the analysis of the Kirchhoff plate model *Continuum Mechanics and Thermodynamics* (2018) doi: <https://doi.org/10.1007/s00161-018-0701-3>.
- [4] Greco, L. and Cuomo, M. and Contrafatto L. Two new triangular  $G^1$ -conforming finite elements with cubic edge rotation for the analysis of Kirchhoff plates, submitted for publication on *Computer Methods in Applied Mechanics and Engineering*