Unstructured T-splines based on local higher-dimensional mesh representations

Roland Maier*, **Philipp Morgenstern**† and **Thomas Takacs**‡

* Institute of Mathematics, Universität Augsburg
Universitätsstr. 14, 86159 Augsburg, Germany
e-mail: roland.maier@math.uni-augsburg.de
web page: https://www.math.uni-augsburg.de/prof/lam/Mitarbeiter/roland_maier/

† Institute of Applied Mathematics, Leibniz Universität Hannover
Welfengarten 1, 30167 Hannover, Germany
e-mail: morgenstern@ifam.uni-hannover.de
web page: https://www.ifam.uni-hannover.de/morgenstern

‡ Institute of Applied Geometry, Johannes Kepler University Linz
Altenberger Str. 69, 4040 Linz, Austria
e-mail: thomas.takacs@jku.at
web page: http://www.ag.jku.at/∼thomast/

ABSTRACT

In 2003, T-splines were introduced in the context of CAD as a new realization for B-splines on irregular meshes that does not require the bookkeeping of a hierarchical basis, but nevertheless allows for local mesh refinement in order to control small-scale geometry features. Shortly after, IGA was introduced, and T-splines were applied with promising results, but were at the same time proven to lack linear independence in certain cases, which actually excludes them from the application in a Galerkin method. Another algorithmic difficulty was revealed in 2011, namely that a naive refinement strategy may not only yield linear dependencies between the shape functions, but also non-nested spline spaces, which compromises the theoretical approximation properties of the method as well as the preservation of exact geometry data during refinement. The issue on linear independence was solved in 2012 with the insight that linear independence is guaranteed if the T-junction extensions do not intersect (analysis-suitability). The second issue, namely how to generate nested spline spaces, was solved in a new refinement algorithm, also preserving linear independence of the T-splines. Still in 2012, new insight on the linear independence of T-splines was provided by introducing the more abstract concept of dual-compatibility and proving its equivalence to analysis-suitability, and in 2013, these concepts were generalized to T-splines of arbitrary polynomial degree, but still restricted to the two-dimensional case. Also at that time, techniques were introduced for the construction of 3D T-spline meshes from boundary representations, motivating the theoretical research on T-splines in three space dimensions, but in particular the linear independence of higher-dimensional T-splines was only characterized through the dual-compatibility criterion, until in 2016, the presenting author of this talk introduced a definition of T-junction extensions and analysis-suitability in three dimensions.

This talk proposes a construction of analysis-suitable T-splines for unstructured meshes with extraordinary nodes of any valence. The construction locally uses higher-dimensional meshes of which the given unstructured mesh is a lower-dimensional projection. We address the local linear independence as well as the local mesh refinement for this kind of T-splines, accounting for the preservation of linear independence and quasi-nestedness of the spline spaces (quasi-orthogonality in the context of mesh-adaptive Galerkin schemes), and sketch ideas towards a theoretical rate-optimality for the application to a second-order elliptic PDE.