CossIGA: using compressive sensing to solve PDEs with isogeometric analysis

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ABSTRACT

Motivated by the difficulties of assembling the Galerkin matrix when using Isogeometric Analysis (IGA) with B-splines of high degree, we propose a novel methodology, called CossIGA (COmpreSSive IGA), which assembles only a small portion of a suitable IGA Petrov-Galerkin discretization of the PDE.

In particular, CossIGA combines IGA techniques with the CORSING (COmpRessed SolvING) method, introduced and studied in [1, 2, 3]. Taking advantage of the compressed sensing paradigm [4, 5], the CORSING method aims at solving efficiently PDEs whose solution is sufficiently sparse (or compressible), i.e., when most of the coefficients of the expansion of the solution with respect to the the basis of trial functions are zero (or negligible). Specifically, after assembling a reduced Petrov-Galerkin system obtained by sampling only a fraction of the rows of the full Petrov-Galerkin matrix, the underdetermined system obtained is then solved by means of sparse recovery techniques (e.g., orthogonal matching pursuit). Since only a fraction of the full matrix needs to be built, this technique could potentially lead to significant computational savings whenever a high-order approximation of a sparse solution is seeked.

In the IGA context, sparsity of the solution is promoted by replacing the B-Spline basis for a given mesh-size h with the union of several such bases generated by L mesh-sizes, i.e. a hierarchical dictionary of B-splines. Our approach can therefore also be seen as connected to the standard h-adaptivity approaches to IGA. We will show some proof-of-concept numerical results that suggest that the proposed approach can be effective in approximating at a reduced computational cost the main features of the PDE solution.

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