

# Efficient Cut-Cell Quadrature for Material Nonlinearity with Application to Tunnel Engineering Simulation

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## ABSTRACT

Subdivision-based quadrature using adaptive quadtree (octree) strategies is robust for a broad class of problems, including plasticity [1, 2], but imposes an excessively high computing cost, especially in 3D. This inherently limits the applicability of cut-cell finite element methods such as the finite cell method [1,2] for practical problems. In this work, the moment-fitting technique [3] that can overcome this drawback is extended to accommodate nonlinear inelastic materials. Using selected fitting functions, quadrature rules defined over an uncut element are constructed to perform accurate integration over the cut one. The effectiveness of this technique lies in the fact that the number of required quadrature points is significantly smaller than in subdivision-based quadrature. The additional computing cost to integrate the fitting functions is a one-time cost, and thus negligible during the whole course of the simulation. To account for material nonlinearity, the cut element is further divided into sub-cells. For each sub-cell, a material point is introduced that is located within the physical domain of the sub-cell, where the constitutive law can be integrated. The integration over the sub-cell is performed by means of the moment-fitting technique. The stress over a sub-cell is considered constant, which facilitates the application of consistent linearization.

The proposed numerical scheme is first successfully validated against well-known finite element benchmarks of Tresca plasticity. To showcase its practical applicability, a machine advancement process in tunnel engineering [4] is then simulated. A B-rep geometry of the excavation volume is embedded into a simple background mesh, which eliminates any additional meshing steps. The surrounding soil is represented as a saturated two-phase medium [5], which consists of the soil matrix and the pore water. The effective stress of the soil with regards to the displacement fields is governed by means of a Mohr-Coulomb model. The results, including the settlement profile and the stress field around the tunnel chamber, are compared with existing results based on standard boundary-fitted finite elements to verify the accuracy of the cut-cell approach with the new quadrature method.

## Keywords

Cut-cell quadrature, moment-fitting, finite cell method, material nonlinearity, tunnel engineering.

## REFERENCES

- [1] D. Schillinger, M. Ruess, The Finite Cell Method: A Review in the Context of Higher-Order Structural Analysis of CAD and Image-Based Geometric Models. *Archives of Computational Methods in Engineering*, Vol. 22, 391-455, (2015).
- [2] A. Abedian, J. Parvizian, A. Düster, E. Rank, The finite cell method for the J2 flow theory of plasticity. *Finite Elements in Analysis and Design*, Vol. 69, p. 37-47, (2013).
- [3] B. Müller, F. Kummer, M. Oberlack, Highly accurate surface and volume integration on implicit domains by means of moment-fitting. *International Journal for Numerical Methods in Engineering*, Vol. 96, p. 512-528, (2013).
- [4] H. G. Bui, A. Alsahly, J. Ninic, G. Meschke, BIM-based Model Generation and High Performance Simulation of Soil-Structure Interaction in Mechanized Tunnelling. *PARENG 2017*.
- [5] F. Nagel and G. Meschke, An elasto-plastic three phase model for partially saturated soil for the finite element simulation of compressed air support in tunnelling. *International Journal for Numerical and Analytical Methods in Geomechanics*, Vol. 34, p. 605-625, (2010).