Spectral analysis for the 2D curl-curl (stabilized) operator with applications to the related iterative solutions

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ABSTRACT

In this work, we focus on large and highly ill-conditioned linear systems of equations arising from different formulations of the Maxwell equations appearing, e.g., in Time Harmonic Maxwell as well as in the MagnetoHydroDynamics, a combination of Maxwell and Navier-Stokes equations used to describe the dynamics of the plasma.

First, we consider a compatible B-Splines discretization based on a discrete De Rham sequence [1] of the 2D curl-curl operator stabilized with zero-order term, and study the structure of the resulting coefficient matrices A_n. It turns out that A_n is a principal submatrix of a two-by-two block matrix where each block is, up to low-rank perturbations, a two-level banded Toeplitz matrix and where the bandwidths grow linearly with the degree of the B-splines.

Moreover, looking at the coefficients of A_n, we identify the so-called *symbol* of $\{A_n\}_n$, a function which compactly describes its asymptotic spectral behaviour. From the knowledge of the symbol and using the theory of the Generalized Locally Toeplitz sequences (see [3]) we show that the matrices A_n are affected by three severe sources of ill-conditioning related to the relevant parameters: the matrix size, the spline degree and the stabilization parameter. As a consequence, when used for solving the associated linear systems, classical methods like the conjugate gradient are extremely slow and their convergence speed is not robust with respect to the parameters.

On this basis, we replace the zero-order stabilization term with a divergence-type one and we compute the symbol of the corresponding Isogeometric discretization matrix sequence. Therefore, we use the retrieved spectral information to design a strategy made up of different basic iterative solvers able to satisfactory deal with all the sources of ill-conditioning. In detail, we use a 2D vector extension of the multi-iterative approach proposed in [2] as preconditioner for the conjugate gradient method and as a result we obtain a computationally attractive and robust solver. Finally, a variety of numerical examples together with some open problems are presented.

REFERENCES

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