# Fast isogeometric finite element method solvers for implicit dynamics

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**ABSTRACT**

The alternating directions method (ADS) was first introduced in [1,2] to deal with finite difference simulations for time-dependent problems. In the ADS method for finite difference simulations, the PDE is discretized using a spatial stencil and intermediate time steps. The direction splitting method has been recently rediscovered by [3,4,5] solve the isogeometric L2 projection problem in the context of isogeometric finite element method. In there, we do not have time steps, but rather the projection problem discretized with tensor products of one-dimensional B-spline basis functions. The direction groups together the one-dimensional B-splines along particular spatial axes. We interpret the mass matrix as a Kronecker product of 1D, multi-diagonal matrices of B-spline basis functions. In this talk, we show how to successfully apply the alternating direction method for isogeometric finite element method simulations of implicit dynamics. Namely, we focus on a parabolic problem and discretize it with B-spline basis functions in each spatial dimension and use implicit scheme for time discretization. We introduce intermediate time steps and separate our differential operator into a summation of the blocks, acting along particular axes in the intermediate time steps. We represent the resulting stiffness matrix as a multiplication of the two (in 2D) or three (in 3D) multiple diagonal matrices, each one encoding the B-spline basis functions in a spatial coordinate. As a result of the algebraic transformations, we factor the system of linear equations in linear (O(N)) computational cost in every time step of the implicit method. Numerical experiments verify our estimates with heat transfer and linear elasticity problems. We conclude our presentation with the discussion on the limitations of the method.

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