Partial tensor decomposition for decoupling isogeometric Galerkin discretisations

Felix Scholz*, Angelos Mantzaflaris* and Bert Jüttler*†

* Radon Institute for Computational and Applied Mathematics
Austrian Academy of Sciences
Altenbergerstraße 69, A-4040 Linz, Austria

† Institute of Applied Geometry
Johannes Kepler University Linz
Altenbergerstraße 69, A-4040 Linz, Austria

ABSTRACT

In isogeometric analysis, tensor decomposition methods can be applied to overcome the computational difficulties when performing the quadrature for assembling the system matrix. Unlike the discretisations used in finite element methods, the spline discretisations that are employed in isogeometric analysis possess a global tensor product structure which can be used in several ways to reduce the complexity of the quadrature. The exploitation of this tensor product structure enables us to deal with the computational disadvantages stemming from the increased polynomial degrees and the larger support of the basis functions.

In [1] a quasi-optimal method for assembling the system matrices in the 2D case using the singular value decomposition (SVD) and the low-rank approximation of the coefficient matrix was introduced. In [2] this was generalised to the case of arbitrary dimensions by performing higher order SVD on the coefficient tensors of the integrands. Again, this leads to a quasi-optimal algorithm. The task of decomposing a tensor of arbitrary order into a sum of outer products is completed by approximating the solution to a nonlinear optimisation problem.

In the present work we avoid performing the full tensor decomposition by only partially decomposing the coefficient tensor of the integrands in a three-dimensional isogeometric discretisation. This means that we use SVD to decompose the trivariate coefficient tensors into tensor products of bivariate and univariate tensors, essentially splitting off one dimension. We observe that this method still results in a quasi-optimal algorithm, because the overall complexity is not governed by the bivariate quadrature but by the following computation of the Kronecker products of the resulting “bivariate” and “univariate” system matrices. For a discretisation with n degrees of freedom and polynomial degree p in each direction, we arrive at an overall complexity of $O(Rn^3p^3)$, where R is the rank of the low-rank approximation needed to fulfil a given error tolerance. In addition, the optimality of the SVD guarantees that the resulting rank value of the low-rank approximation for a given error tolerance is are always less or equal to the rank value in a full tensor decomposition, resulting in even lower computation times.

Our numerical experiments show a large speed up compared to a classical element-wise Gauss quadrature as well as a high stability. The method can be implemented using standard linear algebra tools.

REFERENCES

[1] A. Mantzaflaris, B. Jüttler, B. Khoromskij and U. Langer, “Matrix Generation in Isogeometric Analysis by Low Rank Tensor Approximation”, *Curves and Surfaces*, Springer, pp. 321-340, (2015).

[2] A. Mantzaflaris, B. Jüttler, B. Khoromskij and U. Langer, “Low rank tensor methods in Galerkin-based isogeometric analysis”, *Computer Methods in Applied Mechanics and Engineering*, Vol. 316, pp. 1062–1085, (2017).