

# A nonlinear scaled-boundary isogeometric method for the analysis of solids in boundary representation

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## ABSTRACT

The proposed formulation introduces a numerical method for the nonlinear analysis of solids in boundary representation. The method combines the features of the scaled-boundary approach [1] and the isogeometric analysis [2]. This concept fits perfectly to the surface-oriented modelling of solids in Computer Aided Design Software (CAD). Following the idea of the scaled-boundary finite-element method [1], the geometrical description of the boundary is sufficient to define the boundary value problem for the complete solid. Based on the formulation for the elastic analysis of solids in boundary representation [3], the present approach accounts for material and geometrical nonlinearity. Whereas a nonlinear scaled-boundary-based finite element formulation is provided in [4], the proposed method exploits the tensor-product structure of the solid in order to parameterize the physical domain, i.e., NURBS objects parameterize the two-dimensional boundary surfaces. Additionally, NURBS basis functions are applied in order to approximate the displacement response in the circumferential and radial scaling direction. Hence, the proposed approach is in accordance with the isogeometric paradigm. For material nonlinearity, a small strain theory is assumed. A nonlinear equilibrium equation is introduced due to the nonlinear relation between the stress and the strain. The application of the Galerkin projection of the weak form in the circumferential and radial direction yields a system of nonlinear equations with respect to the unknown displacement response, which is solved by applying the Newton-Raphson scheme. For the geometrical nonlinearity, an elastic material behaviour is assumed. The stress-strain relation is nonlinear with respect to the deformation gradient. The Galerkin projection of the weak form in the circumferential and radial scaling direction leads to a nonlinear equation, whose solution necessitates a linearization. Numerical examples demonstrate the capabilities of the proposed formulation. In the scope of test problems for small strain plasticity and finite strain elasticity, a comparison between the proposed method and the finite-element method are provided.

## REFERENCES

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