BDDC-deluxe preconditioners for isogeometric analysis of linear elasticity problems in three dimensions

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ABSTRACT

Balancing domain decomposition by constraints (BDDC) preconditioners have proven highly effective for the iterative solution of many classes of finite element problems by preconditioned conjugate gradient methods. These algorithms were first introduced by Clark Dohrmann, [4], who also invented its deluxe version which has proven very effective for many problem classes. It has been shown to greatly outperform alternative variants when applied to the often very ill-conditioned systems of algebraic equations which arise in isogeometric analysis.

The goal of the present work is to develop effective and economical BDDC deluxe algorithms for linear elasticity in three dimensions and for isogeometric analysis and to extend the theory previously developed fully only for scalar elliptic problems in the plane, see [1, 2, 3]. In this effort, ideas developed for low order finite element approximations and the closely related FETI–DP algorithms, see [5], have provided guidance. There are several new complications, compared with the finite element work arising from the fact that isogeometric analysis introduces fat interfaces since no workable nodal basis exists for spaces of NURBS functions. The dimension of the Schur complements obtained after eliminating the degrees of freedom associated only with one subdomain is therefore much larger than for nodal finite element methods and an attempt must therefore be made to find effective, small sets of primal constraints. Our work is supported by numerical experiments using software developed using the PETSc programming library.

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