Arbitrary degree, smooth, compatible spline spaces on unstructured quadrilateral and hexahedral meshes

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ABSTRACT

CAD representations of arbitrary genus surfaces with a finite number of tensor-product polynomial patches invariably lead to surface representations with unstructured quadrilateral meshes and, thus, extraordinary vertices. Construction of smooth spline spaces over such unstructured quadrilateral meshes is of considerable interest within the field of isogeometric analysis and a myriad of approaches have been explored that focus on the design and analysis of such geometries. An even bigger challenge lies in the construction of smooth volumetric spline spaces over unstructured hexahedral meshes.

With regard to quadrilateral meshes, a design and analysis methodology for smooth bi-cubic splines on unstructured quadrilateral meshes with extraordinary points was developed in [1] and was observed to be suitable and attractive for applications in both geometric modeling and isogeometric analysis. Singular parameterizations were employed at extraordinary vertices, and the constructed spline spaces demonstrated superior approximation behavior when solving PDEs.

In this work, we present an extension of the methodology developed in [1] from bi-cubic splines to arbitrary odd bi-degree splines on unstructured quadrilateral meshes. Additionally, we construct compatible spaces of discrete differential forms on the quadrilateral meshes and present a high-order, smooth, pointwise divergence-free discretization of the incompressible Navier-Stokes equations.

Finally, utilizing the rigorous approach towards generation of hexahedral meshes for objects of arbitrary genus presented in [2], we extend all our results to smooth trivariate splines defined over unstructured hexahedral meshes.

REFERENCES

[1] Deepesh Toshniwal, Hendrik Speleers, Thomas J.R. Hughes, "Smooth cubic spline spaces on unstructured quadrilateral meshes with particular emphasis on extraordinary points: Geometric design and isogeometric analysis considerations," ICES REPORT 17-05, The Institute for Computational Engineering and Sciences, The University of Texas at Austin, March 2017.

[2] Lei, Na, et al. "Quadrilateral and hexahedral mesh generation based on surface foliation theory." Computer Methods in Applied Mechanics and Engineering (2016).