New Quadrature Rules in Isogeometric Analysis for Symmetric Galerkin Boundary Element Method

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ABSTRACT

In the numerical solution of many differential problems arising in the applications a possible approach is to reformulate the problem by integral equations defined on the boundary of the given domain, giving rise to the so called Boundary Element Methods (BEMs). These methods have two main advantages, the dimension reduction of the computational domain and the simplicity for treating external problems. As a major drawback, the resulting integrals can be singular and therefore robust and accurate quadrature formulas are necessary for their numerical computation. The solution is then obtained by collocation or Galerkin procedures.

With the advent of Isogeometric Analysis (IGA) a new formulation of BEMs has been studied, where the discretization spaces are splines spaces represented in B-spline form. In particular in ([1, 2]) the isogeometric approach has been applied to Symmetric Galerkin BEM (IGA-SGBEM), revealing to be very effective. In order to take all the possible benefits from using B-splines instead of Lagrangian basis, an important issue is the development of specific new quadrature formulas for efficiently implementing the assembly phase of the method. Note that this promising direction has already been taken in the IGA-FEMs context ([4]), but different formulas are needed in the present context because the considered schemes deal with singular or even hypersingular integrals.

In this talk, the problem of constructing appropriate and accurate quadrature rules for SGBEM is presented. Key ingredients are weighted quadrature rules, that are constructed to be exact in the whole test space, also with respect to singular kernel ([3]). Several simulations are presented and discussed, showing accurate evaluation of involved integrals and superiority of the new approach with respect to the standard element-by-element assembly in terms of computational cost and elapsed time.

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