Isogeometric Fup Finite Volume Method for groundwater flow modelling

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ABSTRACT

Groundwater flow modeling, especially in large, highly heterogeneous and anisotropic aquifers, require multi-resolution approach due to resolving many heterogeneity scales, extensive computational resources due to description of large complex domains and accurate velocity field due to solving many important transport analysis in engineering practice such as for instance contaminant pollution. Reliable transport simulations, which support velocity dependent dispersion tensor, require very accurate velocity field which enables mass balance property as well as continuity of velocities and its derivatives. Gotovac et al. [2] developed multi-resolution approach which is based on Fup basis functions and collocation method enabling multiresolution representation of heterogeneity (hydraulic conductivity as a multilevel linear combination of Fup basis functions) as well as all other related variables (such as head or velocity) and accurate, continuous velocity field with all its continuous derivatives. Fup basis functions belong to the class of atomic basis functions and can be regarded as infinitely derivable splines. Particularly, Fup basis function of certain order can be evaluated by convolution of B-spline of the same order and up function. Up function is elementary atomic basis function obtained by infinite convolution of piecewise constant functions where every next convolution function has two times smaller compact support. Since Fup and B-splines enable the same convergence order for smooth problems, Fup basis functions have potentially better approximation properties due to its enhanced continuity. The two main drawbacks of mentioned approach were the application to the only rectangular domains (without description of irregular geometry) and lack of mass balance due to used collocation point method. Therefore, we will present Isogeometric Fup Finite Volume Method (IFFVM) with two main additional ingredients in comparison with mentioned Fup collocation approach: 1) Methodology supports isogeometric analysis (IGA, see for instance [1]) in which geometry, heterogeneity and solution is described with spline basis functions (in this case Fup) in parameter regular domain and 2) Unknown spline (Fup) coefficients are obtained through the weak formulation of the finite volume method [3] using the piecewise constant test functions. The first ingredient enables accurate ("exact" in CAD meaning) description of general complex geometry, while the second one ensures exact mass balance on chosen control volumes which is essentially important for any flow simulation. Since classical IGA uses Galerkin and recently collocation formulations that do not always enable local/global mass balance, finite volume lies between them in sense of convergence and computational cost. Finite volumes are constructed around Greville collocation points and require only integration of fluxes along the finite volume boundaries which is cheaper than full integral Galerkin formulation, but more expensive than point collocation operator evaluation. Moreover, convergence is not optimal as in Galerkin case if appropriate locations and dimensions of finite volumes are not selected which is still open question. Results show heterogeneity influence in terms of log-conductivity variance on convergence and accuracy. *Fup* basis functions and *B* splines are also compared. Methodology is presented and verified using the 2-D numerical and stochastic benchmark simulations.

REFERENCES

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