Accuracy and robustness of Nitsche's method for Dirichlet conditions on small cut elements

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ABSTRACT

CAD geometries in engineering design generally consist of multiple trimmed and overlapping patches [1]. The analysis of such geometries using the framework of isogeometric analysis requires the imposition of coupling and Dirichlet-type boundary conditions [2, 3]. Nitsche's method [4] is a popular approach to impose such conditions in immersed or unfitted finite element methods [5, 6]. From the classical (symmetric) Nitsche's method it is well-known that it requires a sufficiently large stabilization parameter, β, in order to obtain unique solvability of the discrete system. A discrete trace inequality (*e.g.,* [7, 8]) conveys that without additional stabilization techniques β should be inversely proportional to the length scale of the cut element. As cut elements can get arbitrarily small β is in principle unbounded, which deteriorates the mathematical error bounds. This degeneration of Nitsche's method has already been observed in *e.g.,* [9, 2]. In this contribution we describe the theoretical effect on the error bounds and analyze the practical consequenses for the discrete solution. We show in what geometrial configurations these problems can occur such that care has to be taken and discuss possssible resolutions.

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