IGA-based multi-level and multi-index methods for Uncertainty Quantification

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ABSTRACT

In many engineering applications governed by PDEs, the parameters of the equations (coefficients, forcing terms, boundary and initial conditions, shape of the domain) are not known exactly, but rather affected by a certain degree of uncertainties and can be described by means of random variables (or random fields). Uncertainty Quantification (UQ) analysis then aims at estimating how the randomness of these "input" parameters affects the "outputs" of the PDE, typically its solution or functionals thereof.

UQ techniques are often based on repeatedly solving the PDE at hand for different combinations of the input parameters, which requires a significant computational effort. To reduce such effort, so-called "multi-level" and "multi-index" methods have recently been proposed. These methods explore the variability of the PDE outputs using a hierarchy of suitably chosen discretization levels (ideally, to balance the PDE discretization error and the sampling error), and refine the discretization of the PDE only when needed. In this talk, we show how to use these methods in an IGA framework, leveraging on the tensor-structure of IGA solvers.