Determination of superconvergent points for variational collocation with non-uniform B-splines

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ABSTRACT

Isogeometric collocation corresponds to stable one-point quadrature and thus enables a substantial reduction in the cost of formation and assembly. However, for "classical" isogeometric collocation at Greville or Demko points the order of convergence of the solution, measured in the L_2-norm, is known to be sub-optimal.

Recently, Gomez and De Lorenzis [1] introduced the concept of "variational collocation" by showing that, under assumptions which are satisfied in the isogeometric setting, there exist points (called Cauchy-Galerkin points) such that collocation at these points produces the Galerkin solution exactly. Good estimates for Cauchy-Galerkin points were shown to be superconvergent points. These points are about twice more numerous than needed and they had been previously used in [2] within an overdetermined (least-squares) collocation scheme. Using alternately chosen superconvergent points, in [1] the convergence rate of odd-degree discretizations in isogeometric collocation could be improved. Montardini, Sangalli and Tamellini [3] proposed a different selection of superconvergent points and could reach optimality for odd-degree discretizations.

All the previous investigations were limited to uniform meshes, for which case the location of superconvergent points was found. As soon as non-uniform meshes are employed, the location of the superconvergent points changes and must be computed. Herein, we enhance the generality of variational collocation by computing the location of the superconvergent points for non-uniform meshes inspired by the work of Kumar, Kvamsdal and Johannessen [4]. The use of the new points is tested on a number of relevant benchmark cases, where we investigate the performance of the method for different degrees of non-uniformity of the meshes.

REFERENCES

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