# Spline-based Compatible Reduced Basis Methods for Flow Problems

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## ABSTRACT

High-fidelity flow simulations can be quite demanding, involving up to millions or billions of degrees of freedom, and several hours or days of computational time, even on powerful parallel architectures. These techniques become prohibitive when expected to deal quickly and efficiently with repetitive solutions, e.g. in real-time applications. The field of reduced basis models is evolving to address these issues. A reduced basis spans a function space whose dimension is more closely related to the intrinsic physics of the problem, and can be dramatically smaller than those used in high-fidelity simulations, promising potentially real-time performance.

Reduced basis methods for several problems (flow problems among them) are discussed in detail in [1], which is the principal reference. Usage of reduced basis methods for linear and nonlinear flow problems go back several years, see e.g. [2–5]. Their potential for applications in more specific two-dimensional flow problems (cylinders and airfoils) with particular emphasis on wakes are discussed in [6].

In this presentation we employ proper orthogonal decomposition (POD) to construct reduced bases for incompressible cylinder flow problems, where the high fidelity offline simulations are executed on compatible spline spaces. Owing to the continuity condition, the resulting function spaces are divergence-free, a fact which promises certain benefits in the reduced formulation. In particular, we will see that we can make do with a formulation that ignores pressure completely, and still recover it in post-processing for evaluation of functionals such as lift and drag.

## REFERENCES

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