Geometry of the crack-free spherical masonry dome

István SAJTOS*, Orsolya GÁSPÁR, András Á. SIPOS

*Budapest University of Technology and Economics, Department of Mechanics, Materials and Structures
H-1111 Budapest, Műegyetem rkp. 3., Hungary
sajtos@szt.bme.hu

Abstract

Spherical masonry domes are attractive elements of architectural heritage. The spherical form is advantageous in terms of construction and is considered to deliver a spatial message of completeness, eternal power. The often recognizable development of cracks in the meridional direction challenged master builders and later architects and engineers to understand the structural behavior of domes. Membrane theory of shells suggests, that due to the low tensile capacity of masonry, cracks at the lower portion of the dome are inevitable, for the hoop stresses change sign (from compression to tension) considering a hemispherical dome of constant thickness, subjected to its self-weight. Disregarding the limited tensile capacity of masonry, based on a no-tension material model the corresponding extensive literature of the topic offers various theoretical solutions to this problem, which can be classified the following way:

a) the geometry of the middle surface can be altered (e.g. catenarian, or elliptical paraboloid),
b) a thrust surface, different from mid-surface, can be obtained still within the dome section,
c) an optimal shape of spherical domes can be found, which retains the spherical middle surface as membrane surface but allows the thickness of the dome to vary [1], [2].

Present paper offers an extension to the last approach.

It is assumed, that membrane theory is applicable, the middle surface is treated as membrane surface i.e. it is the thrust surface. The membrane shell model of the dome is analogous to the catenary shape masonry arch [3]. The middle surface is hemispherical and it is loaded by the self-weight of the dome. A suitable thickness function is searched, which results a crack-free dome. Such a thickness function is defined, which guarantees compression everywhere or at least zero hoop stresses from top to bottom.

This condition results singularities in the thickness function at the top and at the springing of the masonry dome (i.e. the thickness grows to infinity), which suggest that the result has limited practical relevance. However, interestingly either changing the thickness to a finite value at the top or making an opening at the top (i.e. making an oculus) results compressive hoop stresses all over the spherical dome, thus, the dome remains crack-free. The infinite thickness required at the bottom can also be disregarded, considering a spherical dome with an opening angle slightly lower than 90°.

Historical examples of masonry dome architecture are cited to illustrate the empirical knowledge of the mechanical properties, which is behind the proposed thickness function. It is the aim of present paper to highlight how effective the thickness variation as a structural design tool can be.

References

[1] C. Pesciullesi and M. Rapallini and A. Tralli and A. Chianchi, “Optimal spherical masonry domes of uniform strength”, *Journal of Structural Engineering*, vol. 123, pp. 203-209, 1997.

[2] H. Nakamura, M. Dow, and G. I. N. Rozvany. "Optimal spherical cupola of uniform strenght: allowance for selfweigth", *Ingenieur-Archiv,* 51: (3-4), pp. 159-181, 1981.

[3] S. Adriaenssens, P. Block, D. Veenendaal and C. Williams (eds.), *Shell Structures for Architecture: Form Finding and Optimization*, Routledge, 2014.