Graphic kinematics for 3D frames

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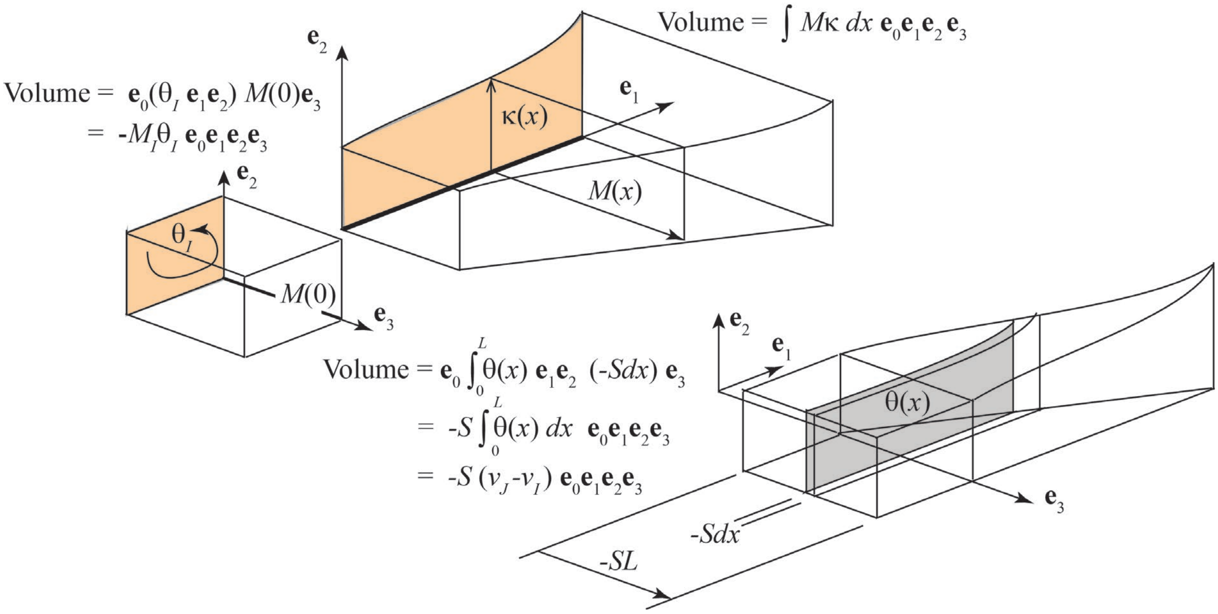
Abstract

For well over a century, the methods of graphic statics have been used to describe the geometric relationship between the shape (form) of a structure and the forces that it carries within it. Historically, the methods began by considering simple 2D pin-jointed trusses loaded at their nodes but have been steadily generalised to apply to a much greater range of structures and conditions, particularly to three dimensions and to moment-resisting frames. One of the most general formulations to date is McRobie's presentation [1] at last year's IASS in Boston. This used a combination of elementary homology theory and Grassmann algebra to construct a complete geometric representation of any structural frame (in 1D, 2D or 3D) and any of its states of self-stress (involving any equilibrium set of axial and two shear forces with torsional and two bending moments).

Complementary to statics is kinematics, and whilst the history of graphic kinematics – the velocity diagrams of mechanism analysis, for example– is of comparable vintage to graphic statics, it is only recently that both the static and kinematic descriptions have been put into a single unifying geometric framework in McRobie *et al.* [2]. There, the Williot-Mohr diagrams of graphic kinematics were related to the 2D Maxwell and 3D Rankine reciprocal form-and-force diagrams of graphic statics. That analysis, though, was only applicable to pin-jointed trusses. The paper here makes the first extensions of that theory to frame structures. The overall geometric setting involves oriented volumes in a pair of dual 4D spaces, and the theory is applicable to frame structures of 1D, 2D and 3D.



References

[1] McRobie A, "A complete graphic statics for self-stressed 3D frames", *IASS2018, MIT Boston*, 2018.

[2] McRobie A, Konstantatou M, Athanasopoulos G and Hannigan L. "Graphic kinematics, visual virtual work and elastographics", *Roy. Soc. Open Sci.*, May 2017, rsos.170202