Solving Large-Scale Truss Layout Optimization Via Semidefinite Programming

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Abstract

The mathematical models in optimization of truss structures are often formulated using the so-called ground structure approach, where a set of nodes is distributed in the design domain and the interconnecting bars are generated. The main goal is to determine the optimal the cross-sectional areas of the bars.

Optimization of truss structures is one of the numerous research disciplines in structural optimization for which a variety of models and methods have been extensively studied. In this study, we address the problem formulations that take into account the global stability of the truss structure resulting in nonlinear and nonconvex semidefinite program. These are in general challenging to solution techniques especially when the size of the problem increases. Thus, existing studies have been considering mostly the minimal connectivity of the bars ignoring most of the potential bars and often are limited to coarse nodal distributions.

We propose a relaxation to the nonlinear models, formulate a (linear) semidefinite program, and describe a primal-dual interior point method and supporting novel techniques to solve these problems efficiently. The method is capable of solving large-scale problems and with full consideration of the potential bars that would be prohibitively difficult for existing solvers. The implementation extensively exploits the sparsity structure and low rank representation of the matrices involved and avoids the bottleneck expensive computational effort to determine the linear systems arising in the interior point algorithm. Moreover, it applies a member adding procedure, which corresponds to column generation in linear programming, to solve smaller sub-problems that ultimately find the solution of the large-scale problem that would otherwise require hundreds of GB storage. Finally, the method employs a warm-start strategy to compute an initial point that reduces the number of interior point iterations to solve some of the subsequent problem instances in the member adding procedure.

We perform several numerical experiments to validate the member adding procedure, show the capability of the interior point method to solve large-scale problem formulations that could never be solved using existing solvers, and finally discuss the validity of the solution to the relaxation formulations.