Numerical Investigation of Stability for Single- and Multi-parameter Systems

Steffen ROTH[*,a], Malte VON SCHEVEN[a], Manfred BISCHOFF[a]

[*,a] Institute for Structural Mechanics, University of Stuttgart
Pfaffenwaldring 7, 70550 Stuttgart, Germany
[*]roth@ibb.uni-stuttgart.de

Abstract

In structural analysis with finite elements, both the solution of a problem (e.g. displacements) and the domain on which it is defined (i.e. the undeformed geometry) are approximated. Spline-based finite elements allow removing the error in geometry approximation also in non-trivial cases by exactly reproducing the geometry, independent of the mesh. For stability analysis of imperfection sensitive structures, slight changes of the initial geometry can tremendously change the results. Here, a perfect initial geometry approximation offers new possibilities because it removes the problem of mesh-dependent, numerical imperfections present in classical finite element analysis. Corresponding effects are demonstrated by means of numerical examples.

In nonlinear stability analysis, computation of the first bifurcation point is significant, e.g. to study imperfection sensitivity or to study the post-buckling behavior. The method of extended systems [1] allows the direct and exact computation of critical points. Here, the required computation of the partial derivative of the stiffness matrix $\mathbf{K}$ with respect to the displacements $\mathbf{d}$ poses a major technical challenge. It can be dealt with by particular numerical differentiation algorithms using complex or hyper-dual numbers, proposed by [2] and [3], which allow computation of exact derivatives. In a further step, the procedure of extended systems is expanded to inhomogeneous Dirichlet boundary conditions ("displacement load cases") and to the case of multiple, independent load parameters.

With the methods described above, buckling sensitive structures can be analyzed efficiently. The extension to multiple load parameters allows the design of motions and to study the stability properties of complex deformation trajectories, for example for deployable and adaptive structures. It thus opens up new possibilities for computational structural design.

References

[1] P.Wriggers, W.Wagner, C.Miehe, A quadratically convergent procedure for the calculation of stability points in finite element analysis, in *Computer Methods in Applied Mechanics and Engineering*, vol. 70, pp. 329-347, 1988

[2] J. R. R. A. Martins, P. Sturdza, J. J. Alonso, The Complex-step Derivative Approximation, in *ACM Trans. Math. Softw.*, vol. 29, pp. 245-262, 2003

[3] J. Fike and J. J. Alonso, The Development of Hyper-Dual Numbers for Exact Second-Derivative Calculations, in *49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, 2011