Geometric approach to form finding of a spoke wheel system: Mathematical explanations

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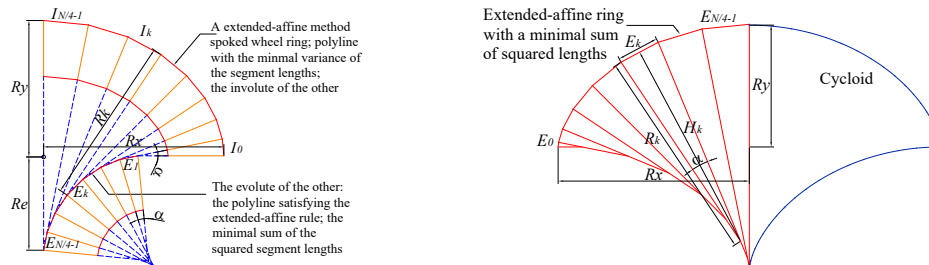Chiyoda-ku Tokyo
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Abstract

The form finding of a spoke wheel roof structure is a highly nonlinear problem as it requires simultaneously determining the geometry of ring and radial cables, flying struts, compression ring as well as ensuring self-stressed equilibrium such that there are only axial forces. While such difficulty lies in the general form of the problem, the affine concept provides a relatively simple approach to determine a circular or oval spoke wheel structure behaving in self-equilibrium. Schlaich Bergermann Partner first applied the concept to a circular roof and later for an oval roof by selecting geometry based on a three-centered arc for the tension and compression rings [1]. However, the application of the concept was limited to a simple and symmetric oval geometry.

The author extended the concept of the affine spoke wheel roof system, not by explicitly defining any circles with different radii but rather implicitly by laying out similar isosceles [2]. The extended concept allows a designer to choose a more arbitrary plan shape to cover with a spoke wheel system and to determine the self-equilibrium geometry and prestresses without sophisticated nonlinear calculations. In fact, any closed convex curve with conditional smoothness can be graphically converted to a self-equilibrium spoke wheel structure.

The investigation of various equilibrium forms generated by the extended-affine concept showed two relations in the geometry of spoke wheel structures in self-equilibrium states. 1) The ring shapes that approximates different ellipses with the minimum variation of segment lengths can always be generated as the involute of a unique polyline. The evolute polyline was found to be a sector of another equilibrium shape that likewise satisfies the extended-affine concept, as well as minimizing the sum of squared lengths of the ring. 2) A numerical comparison indicated that the evolute polyline approaches to a cycloid when the number of segments increases to infinity [2].

This paper revisits these findings in order to give more mathematical explanations and the corresponding drawing steps. Along with the proof of the geometric link, the paper demonstrates how to draw these equilibrium forms without resorting to the nonlinear programming to find the lengths of ring segments.

References

[1] R. Bergermann, K. Göppert, "Das Speichenrad – Ein Konstruktionsprinzip für weitgespannte Dachkonstruktionen", Stahlbau 69, 2000, Heft 8.

[2] H. Tamai, "On a family of equilibrium geometry of a spoke-wheel structure: Geometric approach to form finding of a spoke wheel system", *Creativity in Structural Design: Proceedings of the IASS Annual Symposium 2018*, Boston, USA, July 16-20, 2018.