Finite element methods preserving maximum principles

Santiago Badia*,† and Jesús Bonilla*,†

* Universitat Politècnica de Catalunya
Campus Nord UPC, 08034 Barcelona, Spain
e-mail: sbadia@cimne.upc.edu, jbonilla@cimne.upc,
† International Center for Numerical Methods in Engineering (CIMNE)
Parc Mediterrani de la Tecnologia, Esteve Terradas 5, Castelldefels, Spain

ABSTRACT

Many systems of partial differential equations governing physical phenomena have some underlying structure, e.g., positivity or some maximum principle. However, to inherit at the discrete level such structure is not an obvious task. We are particularly interested in finite element schemes that preserve at the discrete level maximum principles, especially for steady problems and transient problem with implicit time integration. In order to attain such objective, we consider nonlinear stabilization techniques based on a judiciously chosen artificial viscosity. A complete numerical analysis shows that the resulting schemes are discrete maximum princple (DMP) preserving and Lipschitz continuous.

Unfortunately, these additional terms come with a price. The artificial viscosity term is based on a shock capturing detector, which is highly nonlinear and non-differentiable, make extremely hard the nonlinear convergence, and drastically increase the computational cost with respect to non-stabilized formulations. In order to make these algorithms more applicable to real applications, we propose smoothing techniques that lead to differentiable nonlinear viscosity terms, which combined with Newton's method, allows us to clearly improve nonlinear convergence.

The framework has been originally developed for linear and continuous finite element spaces on accute meshes [1], and recently extended to discontinuous Galerkin methods [2] and arbitrary meshes [3, 4]. Since the schemes rely on the convex hull, they loose the DMP property at the discrete level. Finally, in order to go to high order, we will explore the use of B-spline based discretizations.

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