H^1-conforming virtual elements for Darcy and Brinkman equations

L. Beirão da Veiga[†], **C. Lovadina**[‡] **and G. Vacca**[†,*]

[†] Department of Mathematics and Applications, University of Milano Bicocca,
email: lourenco.beirao@unimib.it, giuseppe.vacca@unimib.it

[‡] Department of Mathematics, University of Milano,
email: carlo.lovadina@unimi.it

ABSTRACT

The Virtual Element Method [1] is a generalization of the classical Finite Element Method to arbitrary element-geometry.

In the first part of the talk we present a new family of Virtual Elements for the Stokes problem on polygonal meshes [4]. By a proper choice of the Virtual space of velocities and the associated degrees of freedom, we can guarantee that the final discrete velocity is *pointwise* divergence-free, and not only in a relaxed (projected) sense, as it happens for more standard elements. Moreover, we show that the discrete problem is immediately equivalent to a reduced problem with *fewer* degrees of freedom, thus yielding a very efficient scheme.

The focus of the second part of the talk is on developing a Virtual Element Method for Darcy and Brinkman equations [5]. We use introduce a slightly different Virtual Element space having two fundamental properties: the L^2-projection onto the space of polynomials of degree k (being k the order of the method) is exactly computable on the basis of the degrees of freedom, and the associated discrete kernel is still pointwise divergence-free. The resulting numerical scheme for the Darcy problem has optimal order of convergence and H^1 conforming velocity solution. In particular we obtain a Virtual Element scheme that is accurate for both Darcy and Stokes equations. Then we can apply the same approach to develop a stable virtual element method for Brinkman equations (that is combination of Stokes and Darcy equations).

We provide a rigorous error analysis of the method and several numerical tests, including a comparison with a different Virtual Element choice.

REFERENCES

[1] L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L. D. Marini, A. Russo. Basic principles of virtual element methods. Math. Models Methods Appl. Sci. 23 (2013), 199–214.

[2] B. Ahmad, A. Alsaedi, F. Brezzi, L. D. Marini, A. Russo. Equivalent projectors for virtual element methods. Comput. Math. Appl. 66 (2013), 376–391.

[3] L. Beirão da Veiga, F. Brezzi, L. D. Marini. Virtual elements for linear elasticity problems. SIAM J. Numer. Anal. 51(2) (2013), 794–812.

[4] L. Beirão da Veiga, C. Lovadina, G. Vacca. Divergence free virtual elements for the Stokes problem on polygonal meshes. ESAIM Math. Model. Numer. Anal. (2016)

[5] G. Vacca. H^1-conforming Virtual Element Method for the Darcy equations and the Brinkman equations. In preparation (2016).