DISPERSION-DIFFUSION ANALYSIS FOR ADVECTION PROBLEMS WITH NON-CONSTANT COEFFICIENTS: APPLICATIONS TO DISCONTINUOUS GALERKIN FORMULATIONS

Juan Manzanero*†, Gonzalo Rubio*, Esteban Ferrer*, Héctor R. Díaz-Ojeda§, and Eusebio Valero*

* School of Aerospace Engineering
Universidad Politécnica de Madrid
Plaza del Cardenal Cisneros, 3, 28040, Madrid, Spain

§ School of Naval Engineering
Universidad Politcnica de Madrid,
Avda. Arco de la Victoria 4 28040 Madrid Spain

†e-mail: juan.manzanero@upm.es

ABSTRACT

In this work we study the stability and accuracy of several discontinuous Galerkin (DG) discretizations. The Von Neumann stability analysis is a classic tool to analyse numerical methods. Von Neumann analysis estimates dispersion and diffusion errors introduced by a numerical discretisation by tracking the evolution of wave-like initial conditions when introduced in the numerical scheme. However, the traditional analysis (e.g. [1, 2]) relies on the constant speed advection equation, which may mask features which are common to non-linear problems (i.e. aliasing or propagation speed interpolation errors). For that purpose, we present a non-constant speed Von Neumann stability analysis for the advection equation. This analysis is developed such that it suits the investigation of high order Spectral/hp formulations.

Analysing non-constant advection speed problems allows to discover behaviours from the numerical method, which are not present in the constant case. For example, methods which are stable under constant speed, become unstable for non-constant advection speeds; e.g. DG with Gauss-Lobatto points and central fluxes. This analysis enables the study of new DG formulations, such as the split forms presented in [3, 4], which guarantee stability in aliased conditions.

REFERENCES

[1] G. J. Gassner and D. A. Kopriva, *A comparison of the dispersion and dissipation errors of Gauss and Gauss-Lobatto discontinuous Galerkin spectral element methods*, SIAM Journal on Scientific Computing, 33 (2011), pp. 2560–2579.

[2] R. C. Moura, S. J. Sherwin and J. Peiro, *Linear dispersion-diffusion analysis and its application to under-resolved turbulence simulations using discontinuous Galerkin spectral/hp methods*, Journal of Computational Physics, 298 (2015), pp. 695–710.

[3] G. J. Gassner, *A skew-symmetric discontinuous Galerkin spectral element discretization and its relation to SBP-SAT finite difference methods*, SIAM Journal on Scientific Computing, 35 (2013), pp. 1233–1256.

[4] D. A. Kopriva, and G. J. Gassner, *An energy stable discontinuous Galerkin spectral element discretization for variable coefficient advection problems*, SIAM Journal on Scientific Computing, 36 (2014), pp. A2076–A2099.