Positivity Preserving Limiters for Discontinuous Galerkin Discretizations

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ABSTRACT

In the numerical solution of partial differential equations it is frequently necessary to ensure that certain variables, e.g. density or pressure, remain positive; otherwise unphysical solutions will be obtained that might result in the failure of the numeral algorithm. Positivity of certain variables is generally ensured using positivity preserving limiters, which locally modify the solution to ensure that the constraints are satisfied. For discontinuous Galerkin methods, in combination with explicit time integration methods, this approach works well and many accurate positivity preserving limiters are available, see e.g. [1]. The combination of (positivity preserving) limiters and implicit time integration methods results, however, in serious problems. Many limiters have a complicated, non-smooth formulation that is difficult to linearise, therefore seriously hampering the use of standard Newton methods to solve the nonlinear algebraic equations of the implicit time discretization.

In this presentation, we will discuss a different approach to ensure that the numerical solution satisfies the positivity constraints. Instead of using a limiter, we impose the positivity constraints directly on the algebraic equations resulting from a discontinuous Galerkin method by reformulating the DG equations with constraints using techniques from mathematical optimization theory, [2]. The resulting algebraic equations are then solved using a semi-smooth Newton method that is well suited to deal with the resulting nonlinear complementarity problem [2, 3]. This approach allows the direct imposition of constraints in implicit discontinuous Galerkin discretizations, without the construction of complicated limiters, and results in more efficient solvers for the implicit discretization. We will demonstrate the novel algorithm on a number of model problems relevant for fluid mechanics.

REFERENCES

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