## Implicit Hybrid Upwinding for Multiphase Flow and Transport with Buoyancy and Capillary Pressure

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## Abstract

The partial differential equations governing multiphase flow and transport in heterogeneous porous media are highly nonlinear. Therefore, in the fully implicit finite-volume method, solving the algebraic systems is challenging and accounts for the majority of the simulation cost. We present a numerical scheme applicable to general-purpose simulation that reduces the computational cost by drastically improving the nonlinear convergence.

In the discretized transport problem, the interfacial approximation of the functions of saturation – i.e., phase relative permeability and capillary pressure – has a strong impact on the strength of the nonlinearities. We generalize an approximation method tailored to the multiphase physics and based on Implicit Hybrid Upwinding (IHU) that results in fast nonlinear convergence. This is achieved with a differentiable and monotone numerical flux for two- and three-phase transport obtained from separate evaluations of the viscous, buoyancy, and capillary fluxes. Then, using IHU, we construct an efficient physically based discretization scheme for the mixed elliptic-parabolic problem in which the flow is coupled to the transport of species. Finally, to accurately represent capillary heterogeneity, the proposed scheme accounts for spatially discontinuous capillary pressure functions at the interface between different rock types thanks to discrete transmission conditions.

We present a mathematical analysis that places this new fully implicit finite-volume scheme on a strong theoretical foundation. The mathematical study is supported by challenging heterogeneous two- and three-phase numerical tests demonstrating that the IHU scheme results in significant reductions in the number of nonlinear iterations compared to the commonly used phase-per-phase upstream weighting scheme for viscous-, buoyancy-, and capillary-dominated flow.

## References

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