High order $p-$adaptive DG methods for climate modelling

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ABSTRACT

Discontinuous Galerkin methods have been increasingly applied for the simulation of geophysical flows over the last two decades. High order finite element methods pose however stringent stability restrictions on explicit time discretization methods, thus increasing sensibly the computational cost of their effective large scale application to geophysical problems. Implicit methods have been successfully applied to reduce this cost [1]. In recent work by the authors [2], [3], [4], a comprehensive strategy for the reduction of the computational cost of DG methods for geophysical applications has been proposed. This strategy successfully reduces the computational cost by a combination of two techniques. On one hand, a semi-implicit, semi-Lagrangian time discretization is employed, that allows the use of much longer time steps than explicit schemes while retaining full second order accuracy in time. This approach is complemented by a dynamically adaptive choice of the polynomial degree employed in each element. Results in [2], [3], show that this adaptive strategy allows to reduce the number of degrees of freedom and the associated computational cost by a factor of up to 50%, while retaining the same level of accuracy as the non adaptive discretization. It was then shown in [4] how this approach can be extended to arbitrary unstructured meshes while retaining high order accuracy and mass conservation. In this paper, we will present results of the application of this adaptive technique to idealized high resolution nonhydrostatic flows, that represent typical benchmarks for modern dynamical cores of NWP and climate models. The results show that the method is able to capture phenomena at very different spatial scales and at the same time to reduce significantly the computational cost with respect to standard high order discretizations.

REFERENCES

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