The Harmonic Virtual Element Method and its *hp* version

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ABSTRACT

The Virtual Element Method (VEM) is a recent generalization of the Finite Element Method (FEM), see [1]. The main features of VEM are the employment of polygonal/polyhedral meshes (thus including non conforming meshes) and the possibility of building in an easy fashion global spaces of arbitrary regularity.

In this talk we present a modification of the classical Virtual Element Spaces introduced in [1]. The new method, which takes the name of Harmonic Virtual Element (HVEM), allows for better approximation properties when solving Laplace problems.

The HVEM is inspired by the pioneering work [5] and its main issue is that it employs degrees of freedom only on the skeleton of the polygonal decomposition, thus avoiding to use the unnecessary bubble functions used in standard VEM.

In particular, we describe the *hp* version of HVEM. The idea of *hp* methods is that the convergence of the errors can be achieved by combining properly mesh refinement and increasing the dimension of the local spaces.

Moreover, we compare the *hp* version of HVEM, see [4], with the *hp* version of VEM, see [2, 3], proving at both the theoretical and the numerical level the superiority of the HVEM with respect to the usual VEM, when approximating Laplace problems. In fact, the decay of the error in terms of N, the dimension of the approximation space, is $\exp(-b_1\sqrt[2]{N})$ in the case of *hp* VEM, whereas is $\exp(-b_2\sqrt[3]{N})$ in the case of *hp* VEM.

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