A fully semi-Lagrangian discretization for the 2D Navier-Stokes equations in the vorticity-streamfunction formulation

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ABSTRACT

Semi-Lagrangian (SL) schemes for advection dominated problems have been extremely successful over the last 30 years, allowing for example major reductions in the computational cost of operational weather predictions. Several extensions of SL schemes to diffusion problems have been proposed in the last two decades. A review of these proposals is presented e.g. in [3]. In particular, we will refer to the methods proposed and analyzed in [1], [2], [4]. In these papers, SL methods were shown to be accurate and efficient for linear and nonlinear diffusion problems and for linear advection-diffusion equations. In the present work, we aim to show how a fully SL approach can also be applied to the Navier-Stokes equations, allowing to derive an explicit discretization with very mild stability restrictions that can achieve higher order spatial accuracy in a practically and conceptually simple way, while reducing the computational cost. We present results of a numerical method for the two-dimensional, incompressible Navier-Stokes equations in vorticity-streamfunction form, which employs semi-Lagrangian discretizations for both the advection and diffusion terms, thus achieving unconditional stability without the need to solve linear systems beyond that required by the Poisson solver for the reconstruction of the streamfunction. Several numerical examples validate the proposed method.

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