Certified Reduced Basis Methods for Optimal Control and Data Assimilation

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The reduced basis method is a certified model order reduction technique for the rapid and reliable solution of parametrised partial differential equations, and it is especially suited for the many-query, real-time, and slim computing contexts. This talk begins with a brief introduction into the basic elements of the RB method: approximation, a posteriori error estimation, offline-online computational decomposition, and the greedy algorithm. We then tackle some recent contributions to the field, focusing on problems in optimal control and data assimilation. In particular, we present a certified RB approach to four dimensional variational data assimilation (4D-VAR) [Le Dimet, 1981]. Several contributions have explored the use of reduced order models as surrogates in a 4D-VAR setting (see, for instance, [Stefanescu et al., 2015]). We consider the particular case in which the behaviour of the system is modelled by a parametrised parabolic partial differential equation where the initial condition and model parameters (e.g., material or geometric properties) are unknown, and where the model itself may be imperfect. We consider (i) the standard strong-constraint 4D-Var approach, which uses the given observational data to estimate the unknown initial condition of the model, and (ii) the weak-constraint 4D-Var formulation, which additionally provides an estimate for the model error, and thus can deal with imperfect models. Since the model error is a distributed function in both space and time, the 4D-Var formulation generally leads to a large-scale optimization problem that must be solved for every given parameter instance. We introduce reduced basis spaces for the state, adjoint, initial condition, and model error. We then build upon recent results on RB methods for optimal control problems in order to derive a posteriori error estimates for RB approximations to solutions of the 4D-VAR problem. Numerical tests are conducted to verify the validity of the proposed approach.

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