A Goal-Adaptive Entropy-Stable DG Moment Method for the Boltzmann equation

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ABSTRACT

The Boltzmann equation provides a description of the molecular dynamics of fluid flows based on their one-particle phasespace distribution. The equation is of fundamental importance in a range of high-tech applications, such as semi-conductor photo-lithography devices.

In this presentation we consider a numerical approximation technique for the Boltzmann equation based on a moment-system approximation in velocity dependence and a discontinuous Galerkin finite-element approximation in position dependence. The closure relation for the moment systems derives from minimization of a suitable ϕ-divergence [**?**]. This divergence-based closure yields a hierarchy of tractable symmetric hyperbolic moment systems that retain the fundamental structural properties of the Boltzmann equation. The resulting combined discontinuous Galerkin moment method corresponds to a Galerkin approximation of the Boltzmann equation in renormalized form. The new momentclosure formulation engenders a new upwind numerical flux function, based on half-space integrals of the approximate distribution. We establish that the proposed upwind flux ensures entropy dissipation of the discontinuous Galerkin finite-element approximation [**?**].

By virtue of the hierarchical character of moment-system approximations, the DG moment method is ideally suited to adaptive local refinement based on a-posteriori error estimates. We will present numerical results for goal-oriented adaptive refinement.

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