Flux-corrected transport algorithms for continuous Bernstein finite element approximations

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ABSTRACT

This work extends the flux-corrected transport (FCT) methodology to arbitrary-order continuous finite element discretizations of scalar conservation laws on simplex meshes. Using Bernstein polynomials as local basis functions, we constrain the total variation of the numerical solution by imposing local discrete maximum principles on the Bézier net. The design of accuracy-preserving FCT schemes for high order Bernstein-Bézier finite elements requires a major upgrade of algorithms tailored for linear and multilinear Lagrange elements. In this paper, we propose (i) a new discrete upwinding strategy leading to low order approximations with compact stencils, (ii) a high order stabilization operator based on gradient recovery, and (iii) new localized limiting techniques for antidiffusive element contributions. The optional use of a smoothness indicator based on a second derivative test makes it possible to avoid unnecessary limiting at smooth extrema and achieve optimal convergence rates for problems with smooth solutions. The accuracy of the proposed schemes is assessed in numerical studies for the linear transport equation in 1D and 2D.