Synchronized limiting in high-resolution finite element schemes for the Euler and MHD equations

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ABSTRACT

This talk presents new tools for synchronized limiting of coupled variables in continuous and discontinuous Galerkin methods for systems of conservation laws [1,2]. In the context of the Euler equations, the proposed limiters constrain the gradients of the conservative variables or the corresponding antidiffusive correction terms in a way which guarantees that all quantities of interest remain in the range of admissible values.The bounds for the corresponding inequality constraints are designed to enforce local maximum principles in regions of strong density variations and become less restrictive in smooth regions. The proposed limiting strategy guarantees positivity preservation and leads to closed-form expressions for the synchronized correction factors without the need to solve inequality-constrained optimization problems. In the context of MHD, we consider two predictor-corrector approaches based on the 2D flux-corrected transport algorithm [3]. The first one belongs to the family of unstaggered constrained transport methods using an update of the magnetic vector potential. In the second one,divergence cleaning is performed by using the Raviart-Thomas finite element approximation of the magnetic field. The proposed extensions include the possibility of using nonlinear artificial viscosities based on a splitting of the fastest wave speed in the low-order scheme of Lax-Friedrichs type. The choice of variables to be limited is also discussed and numerical results are presented for standard test problems.

REFERENCES

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