Residual-based variational multiscale modelling of turbulence in an embedded domain DG framework

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ABSTRACT

In the simulation of turbulent flows, the variational multiscale method (VMS) [1] can be used to design fine-scale models that account for the scales not resolved by the finite element discretization. Of particular interest are purely residual-based VMS formulations, where the influence of all unresolved scales is expressed in terms of the residual of the coarse scale equations [2]. One of their advantages, e.g., as compared to VMS formulations with implicit subgrid-scale models [3,4], is a significantly reduced number of degrees of freedom.

Embedded domain Discontinous Galerkin (DG) methods combine the geometric flexibility of unfitted meshes with the advantages of the DG paradigm for high-Reynolds-number flow analysis. Moreover, discontinuous basis functions provide additional flexibility to help mitigate critical problems of embedded domain methods, such as ill-conditioning due to elements with small cuts [5]. A continuous embedded domain method with individual unfitted meshes that are tied together along embedded interfaces can be interpreted as an embedded domain DG method on a macro-element patch level [6].

Purely residual-based formulations have been shown to work very well for continuous finite element schemes [7], but are more intricate for the DG case, where interface terms require careful treatment. Our goal is the consistent integration of a residual-based VMS turbulence model in an embedded domain DG framework. The residual-based VMS approach requires the choice of numerical fluxes for the additional residual terms, for which we discuss different options. Leveraging the analogies in the energy cascades of the Navier-Stokes equations and the Burgers equation, we illustrate its performance for a simple one-dimensional Burgers model. We also discuss the results of initial tests obtained with the resulting framework for high-Reynolds-number flow around an immersed sphere.

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