Efficient quadrature rules for subdivision surfaces in isogeometric analysis

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ABSTRACT

Subdivision surfaces are very popular tools in geometric modeling and have been used recently also in the context of isogeometric analysis [1]. We introduce efficient quadrature rules for spline spaces appearing during the subdivision process. Using the homotopy continuation concept [2] that transforms optimal (Gaussian) quadrature rules from source spaces to target spaces, we derive optimal rules for univariate splines with non-uniform knot vectors. Depending on concrete application such as the Laplace problem and computation of areas or volumes, spline spaces with particular degrees and continuities are considered and optimal univariate rules are derived for them. To derive efficient rules for bivariate splines, we further explore the space of possible element (quad) groupings, and seek the best macro-element segmentation in terms of the number of quadrature points. We show that in the neighborhood of extraordinary vertices (valency other than 4 in the case of quad meshes), where the subdivision spline spaces do not posses the tensor product structure, the most efficient element groupings form $(1 \times n)$ strips. The new rules reduce the number of quadrature points by up to 40% when compared to the traditional Gaussian quadrature.

REFERENCES

[1] Xiaodong Wei, Yongjie Zhang, Thomas J.R. Hughes, and Michael A. Scott. Truncated hierarchical Catmull-Clark subdivision with local refinement. *Computer Methods in Applied Mechanics and Engineering*, 291:1–20, 2015.

[2] M. Bartoň and V.M. Calo. Gaussian quadrature for splines via homotopy continuation: rules for C^2 cubic splines. *Journal of Computational and Applied Mathematics*, 296:709–723, 2016.