A mass, energy, enstrophy and vorticity conserving (MEEVC) mimetic spectral element discretization for the 2D incompressible Navier-Stokes equations

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ABSTRACT

In this work we present a mimetic spectral element discretization for the 2D incompressible Navier-Stokes equations that in the limit of vanishing dissipation exactly preserves mass, kinetic energy, enstrophy and total vorticity on unstructured triangular grids. The essential ingredients to achieve this are: (i) a velocity-vorticity formulation in rotational form, (ii) a sequence of function spaces capable of exactly satisfying the divergence free nature of the velocity field, and (iii) a conserving time integrator. Proofs for the exact discrete conservation properties are presented together with numerical test cases on highly irregular triangular grids.

Extension of these ideas to a spectral element formulation follows by means of the use of tensor products of Lagrange and edge polynomials.

Keywords: energy conserving discretization, mimetic discretization, enstrophy conserving discretization, spectral element method, incompressible Navier-Stokes equations

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