Continuous and Discontinuous FEM-type Approach to Boundary Condition Satisfaction in Vortex Methods for 2D Flow Simulation

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ABSTRACT

In well-known collocation-type discretization schemes it is possible to obtain numerical solution which converges only for average values of solution computed over some parts of the camber line. Convergence rate depends strongly on the shape of the airfoil (is camber line is smooth or it has sharp edges or angular points) and the discretization quality.

Integral satisfaction of boundary condition according to [1] allows to construct much more accurate numerical schemes. If the camber line of the airfoil is approximated by polygon consist of N rectilinear legs ("panels") with maximal length h, when numerical solution is considered to be piecewise-constant on the panels, $O(h^2)$ accuracy can be achieved for average values (C^h-norm), $O(h)$ in L_1-norm and $O(h)$ in C-norm (in case of smooth airfoils and continuous solution).

One of the possible ways to raise the accuracy of vortex sheet intensity computation is Discontinuous Galerkin-type approach. For piecewise linear numerical solution on the panels, it allows to achieve $O(h^2)$ accuracy in L_1, C^h and C (for smooth airfoils) norms. Further improvement of solution in general case is impossible, because the curvature of the camber line should be taken into account.

In all the mentioned schemes linear systems which approximate the corresponding integral equations, are dense, not positively defined and non-symmetric (but well-conditioned), so the commonly used approach for their solution is Gaussian elimination method. Numerical schemes based on DG-approach have significant disadvantage: the dimension of the matrix becomes at least twice bigger ($2N$) than for piecewise-constant solution. Taking into account that in practice when solving coupled FSI problems the value of N can have order of 1 000, and the system changes and should be solved again at every time step, it becomes clear, that such schemes are not good for practice.

In the present research FEM-type approach is implemented and the corresponding numerical schemes for vortex element method is constructed. In the simplest case vortex sheet intensity assumed to be continuous over the camber line and piecewise-linear on the panels. This scheme also provides $O(h^2)$ accuracy in L_1, C^h and C norms, but the linear system has size N. However, this scheme can be applied only for airfoils with smooth camber line, when solution is continuous.

For problems with discontinuous solution, which are very common in aerodynamics, there is only linear convergence in L_1 norm and no convergence in the others. In order to overcome this trouble, the other FEM-type scheme is developed, which permits the discontinuity of the solution in a'priory given points (sharp edges, angular points). The dimension of the matrix becomes $N+p$, where p is number of such points ($p \ll N$). This scheme provides $O(h^2)$ accuracy in L_1 and C^h norms.

For matrix coefficients exact analytical formulae are derived. Some of them are expressed through improper integrals and their numerical computation is hardly possible. In test cases the developed schemes is even slightly more accurate in comparison with DG-scheme with $2N$ matrix size, both for continuous and discontinuous solutions.

REFERENCES

[1] Kempka, S.N. *et al.* Accuracy Considerations for Implementing Velocity Boundary Conditions in Vorticity Formulations. *SANDIA Report SAND96-0583* (1996).