Fluid-structure interaction models in blood microcirculatory districts

Paola Causin*, Gaetano Formato*, Francesca Malgaroli* and Claudio Tocalli*

* Dipartimento di Matematica, Università degli Studi di Milano
via Saldini 50, 20133 Milano, Italy
e-mail: paola.causin@unimi.it

ABSTRACT

Microvessels -blood vessels with diameter less than 200 microns - form large, intricate networks organized into arterioles, capillaries and venules. In these networks, the distribution of flow and pressure drop is a highly interlaced function of single vessel resistances and mutual vessel interactions. Since it is often impossible to quantify all these aspects when collecting experimental measures, in our work we propose a mathematical and computational model to study the behavior of microcirculatory networks subjected to different conditions. The network geometry, which can be derived from digitized images of experimental measures or constructed *in silico* on a computer by mathematical laws, is simplified for computational purposes into a graph of connected straight cylinders, each one representing a vessel. The blood flow and pressure drop across the single vessel, further split into smaller elements, are related through a generalized Ohm's law featuring a conductivity parameter, function of the vessel cross section area and geometry, which undergo deformations under pressure loads. The membrane theory is used for the description of the deformation of vessel lumina, tailored to the structure of thick–walled arterioles and thin–walled venules. In addition, since venules can possibly experience negative values of transmural pressure (difference between luminal and interstitial pressure), a buckling model is also included to represent vessel collapse. The complete model including arterioles, capillaries and venules represents a nonlinear coupled system of PDEs, which is approached numerically by finite element discretization and linearization techniques. As an example of application, we use the model to simulate flow in the microcirculation of the human eye retina, a terminal system with a single inlet and outlet. After a phase of validation against experimental measurements of the correctness of the blood flow and pressure fields in the network, we compute the network response to different interstitial pressure values. Such a study is carried out both for global and localized variations of the interstitial pressure. In both cases, significant redistributions of the blood flow in the network arise, highlighting the importance of considering the single vessel behavior along with its position and connectivity in the network.