Virtual Elements for the Stokes and Navier-Stokes equations

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The Virtual Element Method (in short VEM, introduced in [1, 2]) is recent generalization of the Finite Element Method that enjoys also a connection with modern Mimetic schemes. By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrix, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods, yet still keeping the same coding complexity. For instance, the VEM easily allows for polygonal/polyhedral meshes (even non-conforming) also with non-convex elements and still yields a conforming solution with (possibly) high order accuracy; furthermore, it allows for discrete solutions of arbitrary C^k regularity, defined on unstructured meshes.

In the present talk we introduce the Virtual Element Method in the framework of fluid dynamics, more specifically the Stokes [3] and Navier-Stokes [same group, in progress] equations. We present a method of general order of accuracy that (in addition to enjoying the important advantage of handling general polytopal meshes) exploits the flexibility of Virtual Elements in order to obtain an exactly divergence-free solution. This is well known to yield a set of advantages, when compared to more traditional inf-sup stable methods, that we explore both theoretically and numerically. After a detailed introduction to the method, we present the main theoretical results and close the presentation by showing numerical tests.

References

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[3] L. Beirao da Veiga, C. Lovadina, and G. Vacca. Divergence free virtual elements for the Stokes problem on polygonal meshes. In press on M2AN, DOI: 10.1051/m2an/2016032.