Numerical assessment of a finite element method for viscous flows with stabilized interface pressure jumps

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ABSTRACT

In this work, the two parameter finite element formulation introduced in Buscaglia and Ruas [1] and also studied in [2] is numerically assessed in detail. The finite element method proposed allows to better capture pressure discontinuities at immersed interfaces present in incompressible viscous flows. The Navier-Stokes problem is discretized by means of an equal order formulation, namely the Algebraic Subgrid Scale method, with standard linear triangles for velocity and the space of discontinuous functions proposed in [3] for the pressure field, allowing for arbitrary interface discontinuities. The new ingredient in the formulation being assessed, is an additional stabilizing jump term in those elements cut by the interface that is represented by using the level set method. In the proposed formulation, the pressure jump term, multiplied by a mesh-dependent parameter $\varepsilon \geq 0$ is added to the continuity equation.

Several numerical examples in two spatial dimensions in the axisymmetric case are presented. First, we consider the problem of an actuator disk represented by a planar interface in Poiseuille flow. Second, the problem of a curved interface is studied in a circular Couette flow with moderate although non-negligible inertial effects. Finally, a relevant problem in microfluidics is considered, namely, the thermocapillary (Marangoni) migration of a circular droplet with variable surface tension and known interface curvature. The numerical examples show the superior behaviour of the new formulation for the $L^2(\Omega)$-norm of the pressure error as well as for the error in the pressure jump at the interface. A criterion for the optimal choice of the parameter ε is finally provided.

REFERENCES

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